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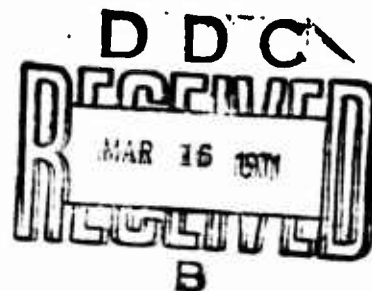
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Source Location with An Adaptive Antenna Array

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NAVAL UNDERWATER SYSTEMS CENTER
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ABSTRACT

An adaptive antenna array processing technique that allows a spatial signal field to be characterized in terms of multiple wavefronts in a background of spatially uncorrelated noise is presented. In particular, the array processor would provide simultaneous, real-time, multisource positional location. The technique is interpreted in terms of an eigenvector decomposition of the single-frequency, spatial-correlation matrix of the data received at the spatial array of sensors. A relationship is established between a multiple wavefront signal field and the eigenvectors of the corresponding single-frequency, sensor-to-sensor, correlation matrix. Because of the orthogonality of the correlation matrix eigenvectors, the processor gives an interference-rejection capability that is useful when locating weak signal sources in the presence of strong interfering sources. Finally, an iterative-adaptive algorithm for the real-time implementation of the array processor is specified.

ADMINISTRATIVE INFORMATION

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SOURCE LOCATION WITH AN ADAPTIVE ANTENNA ARRAY

INTRODUCTION

Current interest in the processing of signals received by antenna arrays has led to a variety of detection and estimation criteria for optimum array signal processing.¹⁻⁴ In accordance with these criteria, several methods for the adaptive realization of the array processing filters have been considered. See, for example, References 5 and 6.

The optimum spatial processing for detecting a distant energy source in a multiple signal field would require one optimum array processor for each possible direction of source location. Thus, if an omnidirectional searching capability is required, the system complexity would increase in proportion to both the required search area and the desired spatial resolution. An exception to this omnidirectional, multibeam array receiver approach that limits the system complexity is discussed by Hyde.⁷ For his technique, the number of sensors (K), rather than the number of search directions, places the primary upper limit on the array processor complexity.

The maximum-likelihood estimation of the location of a signal source with a spatial array of sensors has been considered by several authors.^{4,8,9} Gaarder¹⁰ bypassed the 3-dimensional search requirements of a maximum-likelihood, single-source, location-estimation scheme. He considered easily computed estimates based on an eigenvector decomposition of the sensor-to-sensor wavefront, time-delay correlation matrix.

The approach to multiple-source location taken here is, first, to present an analysis that permits the decomposition of the signal field into orthogonal, eigenvector components. Second, this signal field decomposition is related to a physical model for a multiple-source signal field. Third, the utility of this orthogonal representation is examined. Finally, an iterative adaptive realization for the array processor is described.

A SPATIAL SIGNAL FIELD WITH RESOLVABLE WAVEFRONTS

Consider the K-sensor adaptive array and the M-source signal field shown in Fig. 1. The maximum number of sources (M) is limited to the number of sensors (K). This is because K is the minimum number of resolvable wavefronts that can form a basis for the K-dimensional signal field. The summation of the M wavefronts and an uncorrelated noise component result in a single frequency complex waveform ($X_i(f)$, $i = 1, 2, \dots, K$) at each of the sensor outputs. The i th sensor output is filtered by the adaptively controlled filter $H_i(f)$ and summed with the other filter outputs to form the array processor output ($Y(f)$). The following vector notation is introduced: Let

$$\underline{X} = \begin{bmatrix} X_1(f) \\ X_2(f) \\ \vdots \\ X_K(f) \end{bmatrix} \quad \text{and} \quad \underline{H} = \begin{bmatrix} H_1(f) \\ H_2(f) \\ \vdots \\ H_K(f) \end{bmatrix}, \quad (1), (2)$$

so that the array processor output is given by

$$Y(f) = \underline{H}^T \underline{X}, \quad (3)$$

$$= \underline{X}^T \underline{H}, \quad (4)$$

$$= \sum_{i=1}^K H_i(f) X_i(f). \quad (5)$$

The single frequency, spatial data vector (\underline{X}) can be represented in terms of the superposition of K wavefronts and an uncorrelated noise component as

$$\underline{X} = \sum_{i=1}^K S_i(f) \underline{E}_i(f) + \underline{N}(f), \quad (6)$$

where $S_i(f)$ is the complex envelope for the i th wavefront. The vector

$$\underline{E}_i(f) = \frac{1}{\sqrt{K}} \begin{bmatrix} \exp(-2\pi f r_{i1}) \\ \exp(-2\pi f r_{i2}) \\ \vdots \\ \exp(-2\pi f r_{iK}) \end{bmatrix} \quad (7)$$

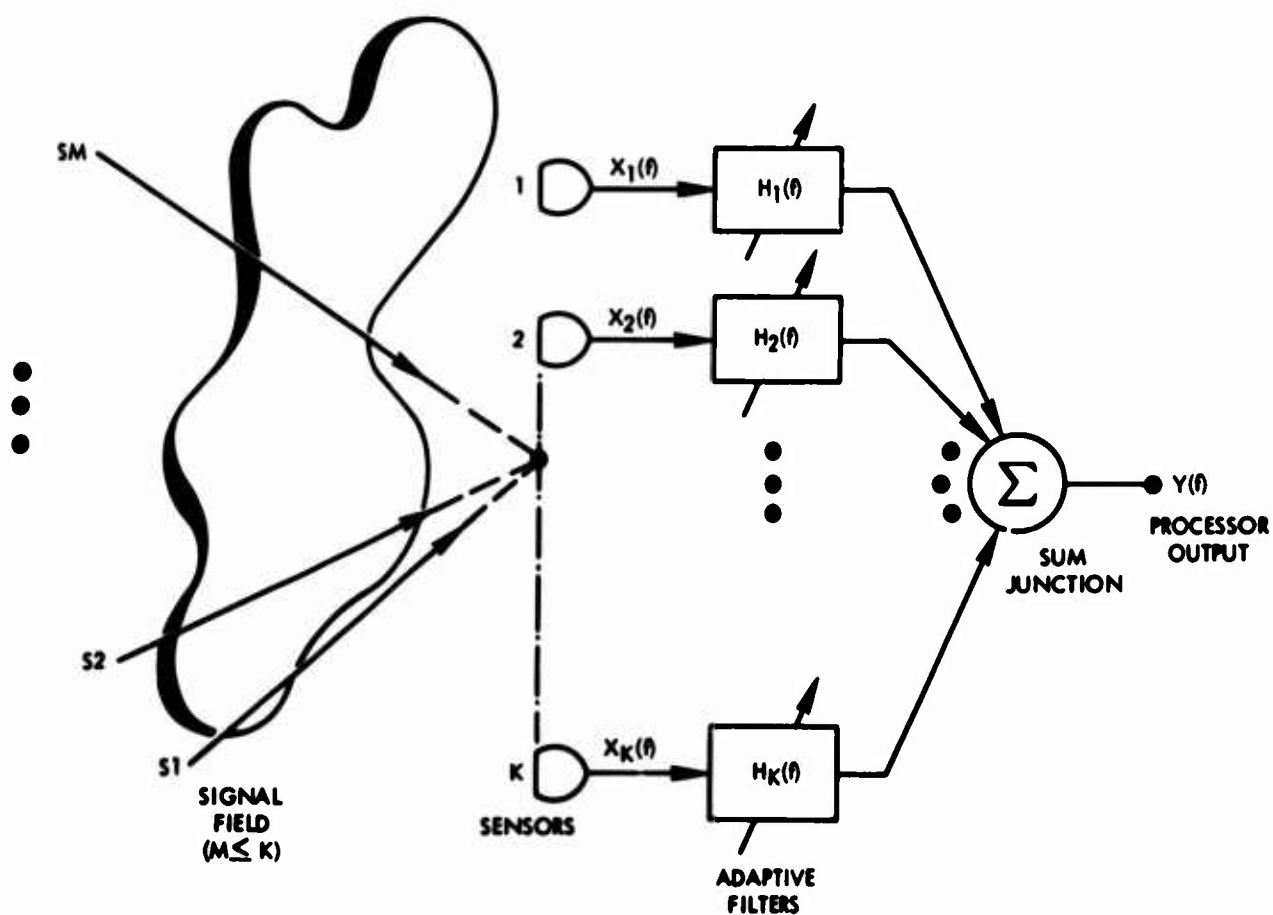


Fig. 1. K-Sensor Adaptive Array in an M-Source Signal Field

is the so-called directional delay vector for the i th wavefront. Note that some of the wavefront complex envelopes ($S_i(f)$) can be zero. Define

$$\underline{N}(f) = \begin{bmatrix} N_1(f) \\ N_2(f) \\ \vdots \\ N_K(f) \end{bmatrix} \quad (8)$$

as the vector of uncorrelated noise components at each of the K sensors. The delay r_{ij} in Eq. (7) is the time required for the i th wavefront to travel to the j th sensor from some arbitrary but fixed spatial reference point.

The single frequency correlation matrix for the received data vector is given by

$$R = \overline{XX^*T} \quad (9)$$

$$= \overline{\left[\sum_{i=1}^K S_i(f) \underline{E}_i(f) + \underline{N}(f) \right] \left[\sum_{j=1}^K S_j(f) \underline{E}_j(f) + \underline{N}(f) \right]^*T} \quad (10)$$

$$= \sum_{i,j=1}^K \overline{S_i(f) S_j^*(f)} \underline{E}_i(f) \underline{E}_j^*(f) + \overline{N(f) N^*(f)} \quad (11)$$

where the overbar represents the time-averaged value. In Eq. (11), the i th wavefront signal envelope $S_i(f)$ is assumed to be uncorrelated with the components in the noise vector $\underline{N}(f)$. Now the matrix \underline{E} is defined with the directional delay vector $\underline{E}_i(f)$ as the i th column. Also, the matrix \underline{P} is defined with ij th element,

$$P_{ij} = \overline{S_i(f) S_j^*(f)} \quad (12)$$

which is the correlation of the complex envelopes of the i th and j th wavefronts. If this notation is employed, Eq. (11) can be rewritten as

$$R = \underline{E} \underline{P} \underline{E}^*T + \underline{P}_0 \underline{I} \quad (13)$$

where

$$\overline{N(f) N^*(f)} = \underline{P}_0 \underline{I} \quad (14)$$

The term " \underline{P}_0 " is the uncorrelated noise power at each of the K sensors and \underline{I} is a $K \times K$ identity matrix.

A statement of wavefront spatial resolvability equivalent to the Rayleigh resolution criterion is desired.^{11,12} In terms of notation previously developed defining the matrix

$$\underline{E} = [\underline{E}_1(f) \underline{E}_2(f) \dots \underline{E}_K(f)] \quad (15)$$

the K wavefronts are spatially resolvable if

$$\underline{E}^*T \underline{E} = \underline{I} \quad (16)$$

In fact, it is sufficient for the largest off-diagonal term in $\mathbf{E}^{*T} \mathbf{E}$ to have a maximum absolute value equal to or less than the inverse of the number of sensors in the array; i.e.,

$$\text{abs} \left[\left(\mathbf{E}^{*T} \mathbf{E} \right)_{ij} \right] = \begin{cases} 1, i = j \\ \leq 1/K, i \neq j \end{cases} \quad (17)$$

for large K .

The following orthogonal forms for the correlation matrices \mathbf{R} and \mathbf{P} are introduced. The spatial correlation matrix \mathbf{R} can be written as

$$\mathbf{R} = \mathbf{M} \boldsymbol{\lambda} \mathbf{M}^{*T}, \quad (18)$$

$$= \sum_{i=1}^K \lambda_i \mathbf{M}_i \mathbf{M}_i^{*T}, \quad (19)$$

where λ_i is the i th eigenvalue of \mathbf{R} , and \mathbf{M}_i is the corresponding eigenvector. Similarly, the complex envelope correlation matrix \mathbf{P} can be written as

$$\mathbf{P} = \mathbf{C} \boldsymbol{\delta} \mathbf{C}^{*T}, \quad (20)$$

$$= \sum_{i=1}^K \delta_i \mathbf{C}_i \mathbf{C}_i^{*T}, \quad (21)$$

where δ_i and \mathbf{C}_i are the corresponding eigenvalue and eigenvector of \mathbf{P} . In Eqs. (18) and (20), the matrices $\boldsymbol{\lambda}$ and $\boldsymbol{\delta}$ are diagonal matrices of eigenvalues for the correlation matrices \mathbf{R} and \mathbf{P} , respectively. In addition, $\boldsymbol{\lambda}$ and $\boldsymbol{\delta}$ are assumed to have the eigenvalues of \mathbf{R} and \mathbf{P} written in monotonically decreasing order on the diagonal. The matrices \mathbf{M} and \mathbf{C} are the so-called modal matrices of \mathbf{R} and \mathbf{P} with columns that are the eigenvectors of \mathbf{R} and \mathbf{P} . These modal matrices have the unitary matrix properties

$$\mathbf{M}^{*T} \mathbf{M} = \mathbf{I} \text{ and} \quad (22)$$

$$\mathbf{C}^{*T} \mathbf{C} = \mathbf{I}, \quad (23)$$

with the result that

$$\mathbf{M}^{*T} = \mathbf{M}^{-1} \text{ and} \quad (24)$$

$$\mathbf{C}^{*T} = \mathbf{C}^{-1}. \quad (25)$$

Thus, by using Eq. (20) in Eq. (13), the correlation matrix R can be written as

$$R = EC\delta C^*E^* + P_0 I \quad (26)$$

Premultiplication and postmultiplication of Eq. (26) by C^*E^* and EC , respectively, yield the diagonal form

$$C^*E^* R EC = \delta + P_0 I, \quad (27)$$

where the identities of Eqs. (16) and (23) have been utilized. Similarly, as in Eq. (18), premultiplication and postmultiplication of R by M^* and M , respectively, yield the diagonal form

$$M^* R M = \lambda. \quad (28)$$

Since the matrices λ and δ represent unique diagonalizations of the matrices R and P , respectively, Eqs. (27) and (28) can be equated with the results

$$\lambda = \delta + P_0 I \text{ and} \quad (29)$$

$$M = EC. \quad (30)$$

Equations (29) and (30) relate the eigenvalues and eigenvectors of the correlation matrix R to the parameters of the spatial signal field, which is given in terms of the wavefront direction matrix E and the complex envelope correlation matrix P . There are three points that are worthy of consideration in the context of the results given by Eqs. (29) and (30):

a. Insight into the bearing response of a conventional array beam-formation system can be obtained;

b. The directional and power-level information for a single resolvable wavefront appears in one eigenvector and eigenvalue, respectively; and

c. A near-optimum adaptive array processing scheme for source localization can be postulated that exploits the wavefront information "clustering" effect alluded to in (b), above.

These three topics are now considered in detail.

CONVENTIONAL ARRAY BEAM FORMATION

A conventional single frequency, time delay-and-sum array beam-steering system is shown in Fig. 2. The time-delay vector for beam steering the array to the bearing angle θ is specified by

$$\underline{D}(\theta) = \begin{bmatrix} D_1(f, \theta) \\ D_2(f, \theta) \\ \vdots \\ D_K(f, \theta) \end{bmatrix} \quad (31)$$

$$= \begin{bmatrix} \exp(-j2\pi f r_1(\theta)) \\ \exp(-j2\pi f r_2(\theta)) \\ \vdots \\ \exp(-j2\pi f r_K(\theta)) \end{bmatrix} \quad (32)$$

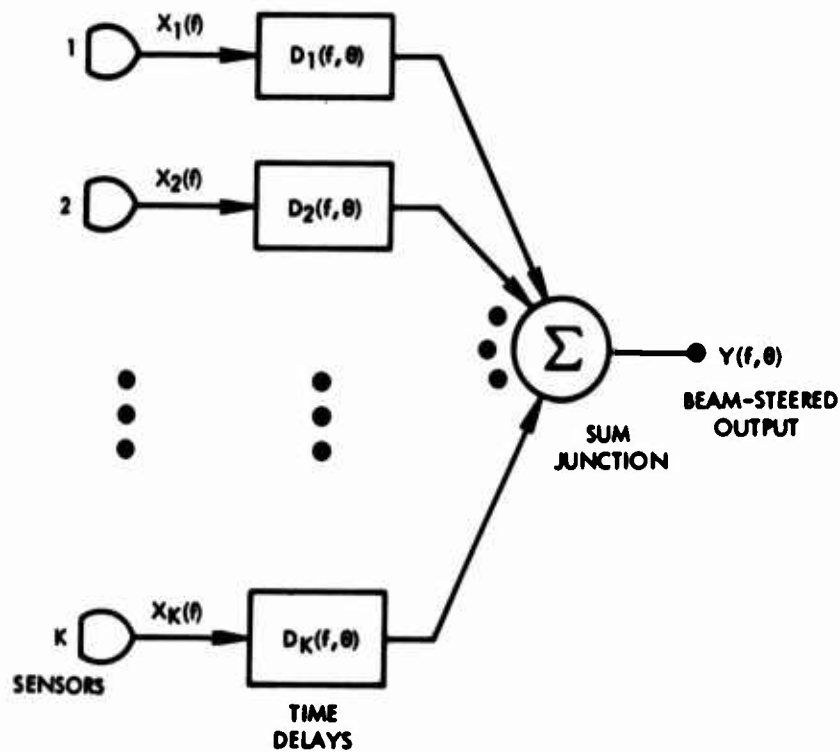


Fig. 2. Single Frequency Time Delay-and-Sum Array Beam Steering

The time delay $\tau_i(\theta)$ is the delay inserted at the i th sensor to steer a beam from the array to a plane wave coming from direction θ . Therefore, the beam-steered output ($Y(f, \theta)$) can be specified by

$$Y(f, \theta) = \underline{D}^T(\theta) \underline{X}. \quad (33)$$

The bearing response $B(f, \theta)$ for the array that is beam steered at angle θ is given by

$$B(f, \theta) = \overline{|Y(f, \theta)|^2}. \quad (34)$$

$$= \underline{D}^T(\theta) \underline{X} \underline{X}^{*T} \underline{D}^*(\theta), \quad (35)$$

$$= \underline{D}^T(\theta) \underline{R} \underline{D}^*(\theta), \quad (36)$$

$$= \underline{D}^T(\theta) \underline{M} \underline{\Lambda} \underline{M}^{*T} \underline{D}^*(\theta), \quad (37)$$

$$= \underline{D}^T(\theta) \sum_{i=1}^K \lambda_i \underline{M}_i \underline{M}_i^{*T} \underline{D}^*(\theta), \quad (38)$$

$$= \sum_{i=1}^K \lambda_i |\underline{D}^T(\theta) \underline{M}_i|^2, \quad (39)$$

$$B(f, \theta) = \sum_{i=1}^K \lambda_i B_i(f, \theta), \quad (40)$$

where

$$B_i(f, \theta) = |\underline{D}^T(\theta) \underline{M}_i|^2 \quad (41)$$

is defined herein as the i th eigenvector bearing response.¹³ The response $B_i(f, \theta)$ is interpreted with reference to Fig. 1 by letting $\underline{H} = \underline{M}_i$ and observing that $B_i(f, \theta)$ is the array processor output power for a unit power source, which is generating a plane wave at the array from bearing angle θ . Thus, the conventional beam formation bearing response has been decomposed into the sum of K eigenvector bearing responses, each weighted by the appropriate eigenvalue.

The importance of Eq. (40) in obtaining $B(f, \theta)$ is that the complete bearing response can be obtained directly from a measurement of the eigenvalues and vectors of the correlation matrix R . This approach is in contrast to sampling the spatial field with beams that have finite spatial beamwidth and beam-to-beam spacing. However, this interpretation is only one rationale for decomposing $B(f, \theta)$. The real importance of Eq. (40) lies in the actual distribution of spatial-field information in the λ_i and $B_i(f, \theta)$ terms. This effect is considered in detail in the following section.

SPATIAL SIGNAL FIELD DESCRIPTION BY EIGENVECTORS

For K resolvable signal wavefronts with statistically uncorrelated complex envelopes, the wavefront correlation matrix P is diagonal. For this case, it follows from Eqs. (29) and (30) that

$$\lambda = KP + P_0 \quad \text{and} \quad (42)$$

$$M = E, \quad (43)$$

with the i th eigenvalue given by

$$\lambda_i = KP_{ii} + P_0, \quad (44)$$

where $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_K$. The corresponding eigenvector is

$$E_i = E_i(f), \quad (45)$$

where P_{ii} is the power in the i th signal wavefront and $E_i(f)$ is the wavefront delay direction vector ($i = 1, 2, \dots, K$). Thus, the conventional bearing response according to Eq. (40) is

$$B(f, \theta) = \sum_{i=1}^K (KP_{ii} + P_0) |\underline{D}^T(\theta) E_i(f)|^2, \quad (46)$$

where the i th eigenvector bearing response, Eq. (41), is given by

$$B_i(f, \theta) = |\underline{D}^T(\theta) E_i(f)|^2, \quad (47)$$

Equation (47) has an absolute maximum at θ_i , where θ_i is the bearing angle for a farfield point signal source. The importance of this result is that the bearing response $B_i(f, \theta)$ is invariant with respect to the signal-to-noise power ratio (S/N) for the j th wavefront,

$$(S/N)_j = P_{jj}/P_o \quad (48)$$

where $j = 1, 2, \dots, K$, and, in fact, depends only on the directional properties of the i th wavefront. Therefore, in theory at least, the localization of the i th source by utilizing $B_i(f, \theta)$ is independent of relative signal levels. Alternatively, locating the i th source by means of Eq. (47) allows effective discrimination against all other spatially resolvable wavefronts regardless of the power of that wavefront. This effect is illustrated in the following example: A line array with 24 sensors is considered. The sensors are spaced one half a wavelength ($\lambda/2$) apart with respect to two random, uncorrelated signal sources at frequency f . Figure 3 illustrates the array and signal-field parameters. Figures 4 and 5 show the normalized angular bearing response (Eq. (47)) for eigenvectors $\underline{M}_1 = \underline{E}_1(f)$ and $\underline{M}_2 = \underline{E}_2(f)$, respectively. It is important that, in view of the fact that source S2 is 20 dB below source S1 in S/N , the bearing response for \underline{M}_2 , when compared with that for \underline{M}_1 , is in no way degraded with respect to locating source S2. Moreover, it is easy to observe a notch in the \underline{M}_2 response sidelobe region in the vicinity of S1. This is because \underline{M}_2 performs spatial filtering on the received data vector X , which rejects the S1 wavefront contained in the orthogonal eigenvector \underline{M}_1 . This effect occurs because $\underline{E}_1^* \underline{E}_2 \neq 0$; i. e., the sources are not ideally resolvable according to Eq. (16). However, Eq. (17) is not violated for this example, and the orthogonality of eigenvectors \underline{M}_1 and \underline{M}_2 containing source locations θ_1 and θ_2 provides spatial sidelobe discrimination.

Figure 6 shows the conventional bearing response (Eq. (40)) truncated after the first two terms; i. e.,

$$\hat{B}(f, \theta) = \sum_{i=1}^2 \lambda_i B_i(f, \theta) \quad (49)$$

Clearly, high level source S1 has suppressed the spatial recognition of source S2 owing to the relative difference in S/N . In fact, if the conventional bearing response were computed according to Eq. (40), source S2 would be subordinated to a -20 dB bearing-response level with respect to S1. This degree of suppression of the S2 bearing-response level is in contrast to the -13 dB suppression shown in Fig. 6 and the complete independence of θ_2 via $B_2(f, \theta)$ on relative source S/N illustrated in Fig. 5.

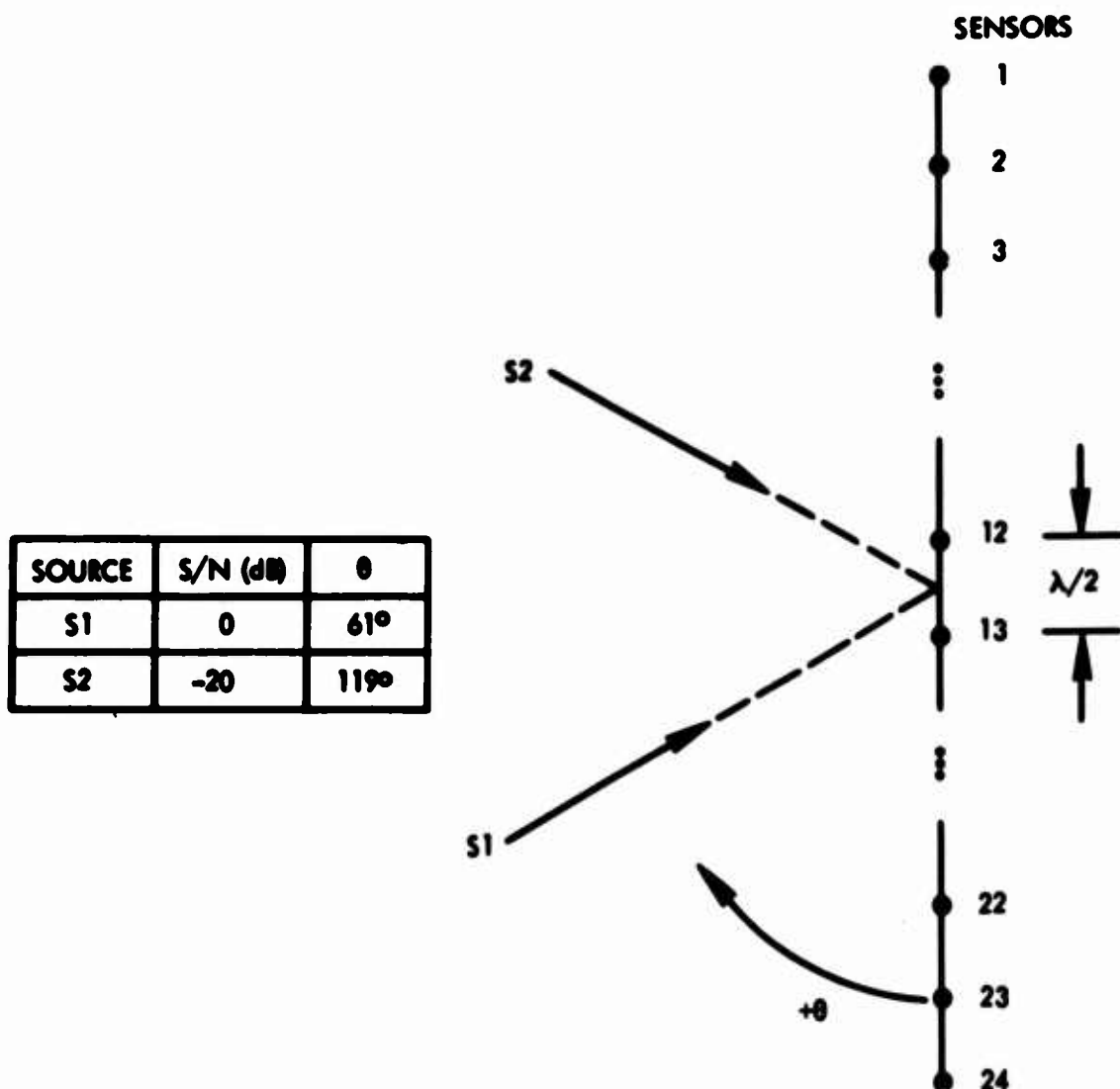


Fig. 3. A $K=24$ Sensor Line Array in a Spatial Signal Field Consisting of Two Uncorrelated Sources at Frequency f

For correlated wavefront complex envelopes, Eq. (30) indicates that directional information on all targets may appear in any given eigenvector, as determined by the modal matrix C . It is shown by means of an example in the following section that the signal-related information in a highly correlated multisource environment appears in the first eigenvector.

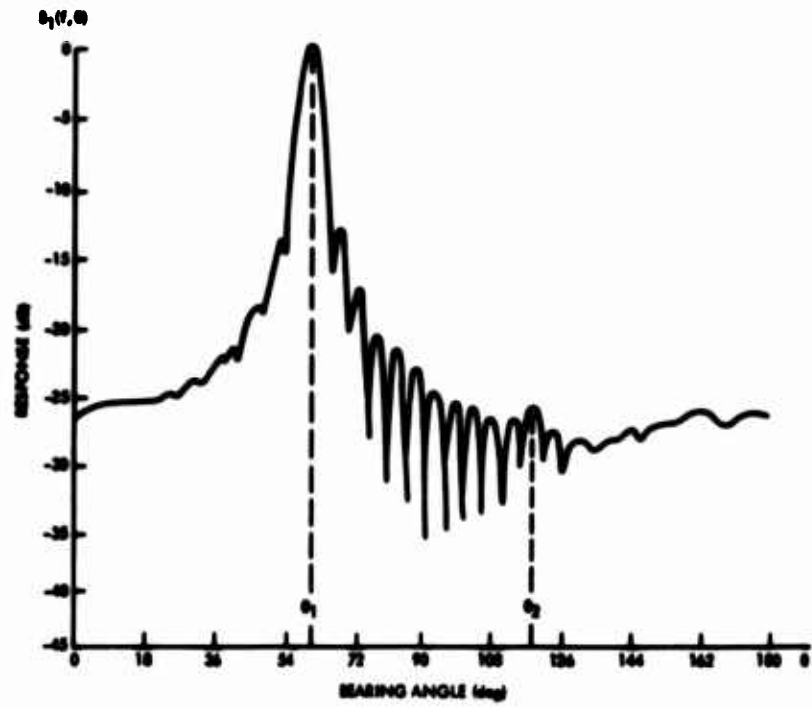


Fig. 4. Bearing Response for Eigenvector M_1 ($B_1(t, \theta) = |D^T(\theta) M_1|^2$)

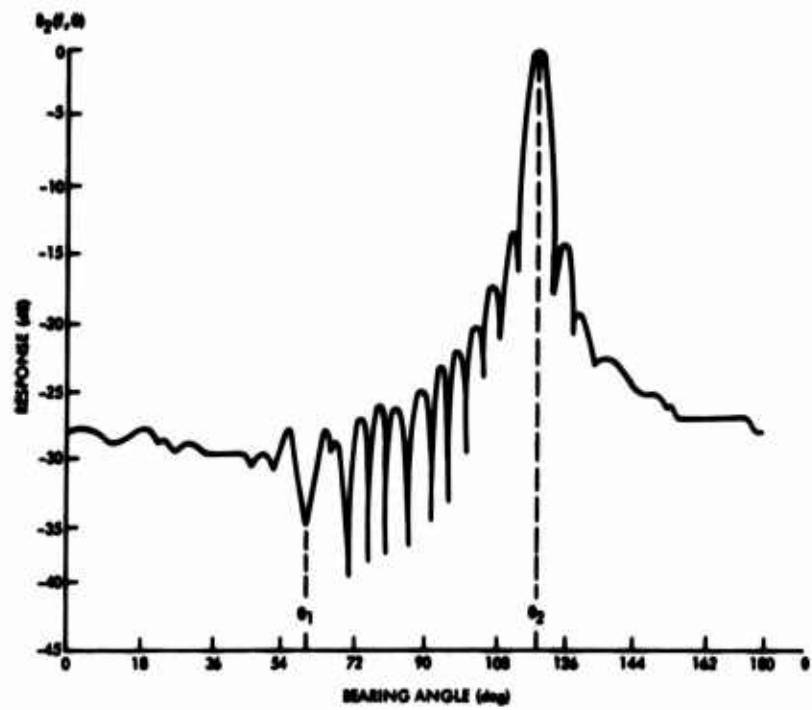


Fig. 5. Bearing Response for Eigenvector M_2 ($B_2(t, \theta) = |D^T(\theta) M_2|^2$)

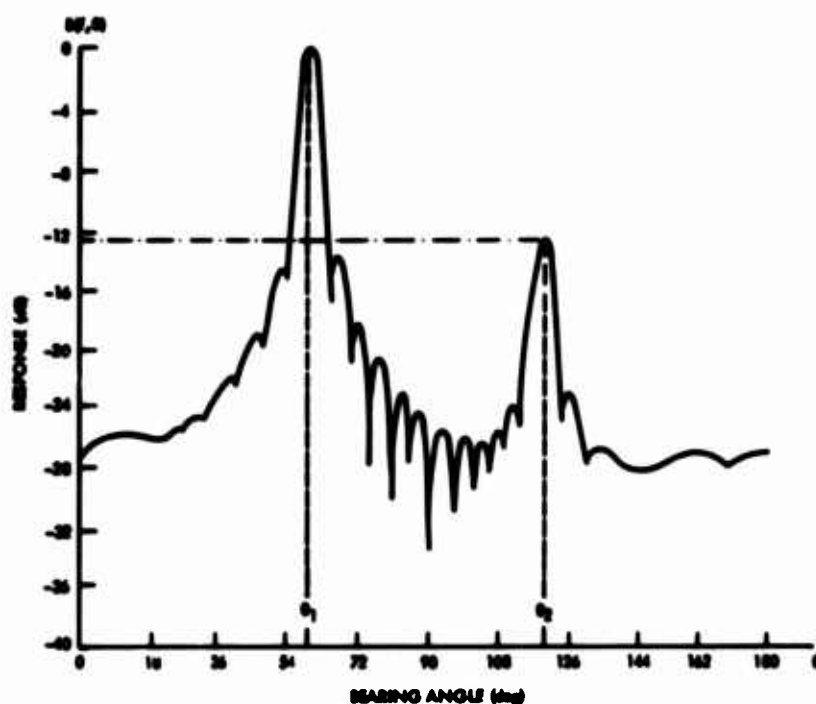


Fig. 6. Two-Term Truncated Approximation of the Conventional Bearing

$$\text{Response } (B(f, \theta)) = \sum_{i=1}^K \lambda_i |D^T(\theta) M_i|^2$$

ADAPTIVE REALIZATION FOR MULTIPLE SOURCE LOCATION ARRAY PROCESSOR

In the previous section it was shown that resolvable multi-wavefront information appears in the eigenvalues and associated eigenvectors of the single frequency, sensor-to-sensor, array correlation matrix R . Applications of eigenvector analysis, in terms of the Karhunen-Lóeve expansion, to the analysis of random process sampled data are well known.¹⁴⁻¹⁷ In particular, Karhunen-Lóeve expansions find utility in signal processing^{14,17,18} and in the slightly more general areas of pattern recognition and dimensionality reduction.^{15,16,19}

To date, the numerical techniques for determining eigenvalues and associated eigenvectors of an estimated correlation matrix of observed random data have required a large computational storage area and have not been feasible in real time. Briefly, existing techniques require, first, the estimating of the covariance matrix of dimension $(K \times K)$ for K -dimensional sample vectors. Generally, these estima-

tion procedures utilize a "lagged-product" approach, where time-delayed sample products are averaged to estimate the correlation elements of the matrix. This method implicitly assumes interval stationarity of the time-sampled data. Second, subsequent to the correlation matrix estimation, the computationally most expedient techniques utilize variations of Jacobi's method to compute eigenvalues and eigenvectors from the estimated matrix.^{19,20,21} Even though such methods are recursive, considerable computer storage space, word handling, and computation time are required.

The method discussed herein presents a numerical algorithm for eigenvalue-eigenvector estimation based on a constrained gradient search method.²²⁻²⁵ This technique has the following desirable features:

a. Minimal storage requirements, proportional to (KXM) , where K is the sample vector dimensionality and M is the number of eigenvalues (vectors) that are to be obtained.

b. A potential for real-time operation.

Basically, the constrained gradient search (CGS) algorithm applied to eigenvalue (vector) estimation provides for a sequential estimation of eigenvalues and associated eigenvectors in order of decreasing eigenvalue magnitude. A matrix form of the Hamiltonian criterion function that is amenable to maximization via gradient search and stochastic estimation is central to the development.

Consider the Hermitian form

$$P_{y1} = \underline{H}_1^T \left(\underline{R} - \sum_{j=1}^{M-1} \lambda_j \underline{M}_j \underline{M}_j^{*T} \right) \underline{H}_1^* , \quad (50)$$

$$= \underline{H}_1^T \sum_{j=1}^K \lambda_j \underline{M}_j \underline{M}_j^{*T} \underline{H}_1^* , \quad (51)$$

$$= \sum_{j=1}^K \lambda_j |\underline{H}_j^T \underline{M}_j|^2 , \quad (52)$$

where it is recalled that \underline{R} is the correlation matrix for the input sampled data vector, which can be written as in Eq. (19) to yield Eq. (52). It can be shown,²⁶ providing that for $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_K$, P_{y1} is a maximum for \underline{H}_1 , subject to the constraint

$$\underline{H}_i^T \underline{H}_i^* = |\underline{H}_i|^2 \quad (53)$$

$$= 1, \quad (54)$$

if $\underline{H}_i = \underline{M}_i^*$ and $P_{yi} = \lambda_i$. That is,

$$\max_{|\underline{H}_i|^2 = 1} \{P_{yi}\} = \lambda_i \quad (55)$$

for $\underline{H}_i = \underline{M}_i^*$. Thus, if the first $(i-1)$ eigenvalues and corresponding eigenvectors of the correlation matrix \underline{R} are available, the i th eigenpair, λ_i and \underline{M}_i , can be obtained by invoking Eq. (55) and searching for an \underline{H}_i , which maximizes P_{yi} .

For the present application, a numerical search for \underline{H}_i can be implemented. For this purpose, the method of undetermined Lagrange multipliers is utilized to obtain an orderly search procedure. See, for example, Reference 25, Chapter 13, and Reference 27.

Suppose the sample vector $\underline{X}(n)$ of the vector random process \underline{X} in Eq. (1) is available at time instant t_n . If the value of the filter vector in Eq. (2) at t_n is $\underline{H} = \underline{H}_i(n)$, then $\underline{H}_i(n)$ is to be selected in order to maximize

$$P_{yi}(n) = \underline{H}_i^T(n) \left[\underline{R} - \sum_{j=1}^{i-1} \lambda_j \underline{M}_j \underline{M}_j^{*T} \right] \underline{H}_i^*(n) \quad (56)$$

at instant t_n subject to the constraint

$$|\underline{H}_i(n)|^2 = 1. \quad (57)$$

Application of the method of undetermined Lagrange multipliers stipulates that the Hamiltonian,

$$h_i(n) = P_{yi}(n) + L[1 - |\underline{H}_i(n)|^2], \quad (58)$$

must be an extremum with respect to $\underline{H}_i(n)$ as a necessary condition for maximizing $P_{yi}(n)$, subject to the constraint of Eq. (57). In Eq. (58), L is the undetermined multiplier that is required for the magnitude constraint on $\underline{H}_i(n)$. Accordingly, the gradient of $h_i(n)$, with respect to $\underline{H}_i(n)$, is given by

$$\nabla_{\underline{H}_i(n)} h_i(n) = 2 \left[\underline{R} - \sum_{j=1}^{i-1} \lambda_j \underline{M}_j \underline{M}_j^{*T} - L \underline{I} \right] \underline{H}_i^*(n). \quad (59)$$

Solving Eq. (59) at the zero gradient point yields

$$\left[R - \sum_{j=1}^{I-1} \lambda_j \underline{M}_j \underline{M}_j^{*T} \right] \underline{H}_1^*(n) = L \underline{H}_1^*(n) , \quad (60)$$

which is an eigenvalue matrix equation form. Premultiplication of Eq. (61) by $\underline{H}_1^T(n)$ yields

$$\underline{H}_1^T(n) \left[R - \sum_{j=1}^{I-1} \lambda_j \underline{M}_j \underline{M}_j^{*T} \right] \underline{H}_1^*(n) = L \underline{H}_1^T(n) \underline{H}_1^*(n) , \quad (61)$$

$$= L |\underline{H}_1(n)|^2 , \quad (62)$$

$$= L , \text{ and} \quad (63)$$

$$= P_{y1}(n) . \quad (64)$$

Thus, the undetermined Lagrange multiplier (L) is in fact an eigenvalue of the residue matrix

$$\left[R - \sum_{j=1}^{I-1} \lambda_j \underline{M}_j \underline{M}_j^{*T} \right] = \sum_{j=1}^K \lambda_j \underline{M}_j \underline{M}_j^{*T} . \quad (65)$$

Furthermore, L is equal to λ_1 , which is the maximum eigenvalue in Eq. (65), because $P_{y1}(n) = L$ is being maximized. Therefore, the vector $\underline{H}_1(n)$, which maximizes P_{y1} , is the eigenvector \underline{M}_1 .

Now an iterative filter adjustment algorithm for determining $\underline{H}_1(n) = \underline{M}_1^*$ can be specified by using the method of steepest gradient ascent.^{22,23} For this method, we adjust $\underline{H}_1(n)$ according to

$$\underline{H}_1(n+1) = \underline{H}_1(n) + \mu \underline{J}_1(n) , \quad (66)$$

where $\underline{J}_1(n)$ is an estimate of the gradient $\nabla_{\underline{H}_1(n)} h_1(n)$ that is given by

$$\underline{J}_1(n) = 2 \left[R - \sum_{j=1}^{I-1} \lambda_j \underline{M}_j \underline{M}_j^{*T} - \hat{L} \mathbf{I} \right] \underline{H}_1^*(n) , \quad (67)$$

$$\underline{J}_1(n) = 2 \left[\underline{X}(n) \underline{X}(n)^{*T} - \sum_{j=1}^{I-1} \lambda_j \underline{M}_j \underline{M}_j^{*T} - L(n) \mathbf{I} \right] \underline{H}_1^*(n) , \quad (68)$$

$$= 2 \left[\underline{X}(n) \underline{X}(n)^{\circ T} \underline{H}_i^{\circ}(n) - \sum_{j=1}^{i-1} \lambda_j \underline{M}_j \underline{M}_j^{\circ T} \underline{H}_i^{\circ}(n) - L(n) \underline{H}_i^{\circ}(n) \right], \quad (69)$$

$$= 2 \left[\underline{X}(n) Y_i^{\circ}(n) - \sum_{j=1}^{i-1} \lambda_j \underline{M}_j \underline{f}_{ji}^{\circ}(n) - L(n) \underline{H}_i^{\circ}(n) \right]. \quad (70)$$

In Eq. (67), the estimates

$$\hat{\underline{R}} = \underline{X}(n) \underline{X}^{\circ T}(n) \quad \text{and} \quad (71)$$

$$\hat{\underline{L}} = P_{y_i}(n), \quad (72)$$

$$= \overline{|Y_i(n)|^2}, \quad (73)$$

$$= \overline{|\underline{H}_i^T(n) \underline{X}(n)|^2} \quad (\text{See Eq. (64).}) \quad (74)$$

are utilized. The constant μ in Eq. (66) is a small positive quantity that controls the rate of convergence of $\underline{H}_i(n)$ to \underline{M}_i° and the variance of the convergent solution. Finally, the filter-control algorithm, called the λ_i algorithm, given by Eqs. (66) and (70), specify the array processor. Operationally, the processor would allow the filter vector to converge to \underline{M}_i° and then compute an angular bearing response according to Eq. (41). Either this bearing response could be displayed or the peak values of the response could be retained as an estimate of the bearing angle for the source(s) appearing in eigenvector \underline{M}_i .

With regard to storage requirements for implementing this processor, consider Eqs. (66) and (70). The first $(i-1)$ eigenvectors must be stored for each λ_i algorithm. In addition, a set of update storage registers must be available for the $\underline{H}_i(n)$ vectors. Note, however, that neither is there a requirement to store a complete estimate of the correlation matrix \underline{R} nor is there a necessity to call a computer subroutine for computing the desired eigenvalues and vectors. For arrays with a large number of sensors and a broad spectrum of interest, these two computational factors are of considerable importance from an implementation standpoint. This is particularly true if the desired number of eigenpairs (M) is much less than the number of sensors in the array (K).

A verification of the convergence of the λ_1 algorithm can be obtained if the time-averaged array processor output power converges to the maximum eigenvalue (λ_1). An estimate of this maximum eigenvalue is computed by estimating the correlation matrix and then by performing an eigenvalue decomposition. An additional verification of convergence is obtained by computing the eigenvector corresponding to λ_1 by means of an eigenvector subroutine and then by employing this vector to specify the array bearing-response pattern. The resultant pattern should correspond to that obtained when the adaptive λ_1 algorithm approach is used. Examples of the above-mentioned verifications follow.

The λ_1 algorithm given by Eq. (66) is employed to locate simultaneously three nonrandom, single-frequency energy sources of initially unknown angular positions with respect to the longitudinal axis of a 24-element linear array. (See Fig. 3.) Uncorrelated noise is inserted at each sensor of the adaptive array. These sensors are spaced one half a wavelength apart. The filter vector $\underline{H}_1(0)$ is initialized with weights that give an omnidirectional sensitivity pattern, i.e., equal sensitivity in all directions, so as not to initially bias the adaptive array in a particular direction. The sources located by the adaptive array processor are spatially stationary and exhibit S/N of -12, -3, and -3.6 dB for sources S1, S2, and S3, respectively.

Figure 7 gives the single frequency sensitivity patterns for the adaptive and maximum eigenvalue weight vectors. The two independently obtained sensitivity patterns are in close agreement. Figure 8 gives the adaptive processor's "learning" curve, which is the time-averaged output power of the adaptive array processor versus number of filter adaptations. An independent calculation of the maximum eigenvalue from an estimate of the correlation matrix \hat{R} is in close agreement with the adaptive array processor output power subsequent to convergence. This comparison is given by

$$10 \log \lambda_1 = 13.50 \text{ dB} \quad \text{and}$$

$$\lim_{n \rightarrow \infty} \overline{|Y_1(n)|^2} = 13.60 \text{ dB} ,$$

where the overbar indicates time averaging. For this example, 10,000 iterations of Eq. (66) were implemented and a value of μ equal to

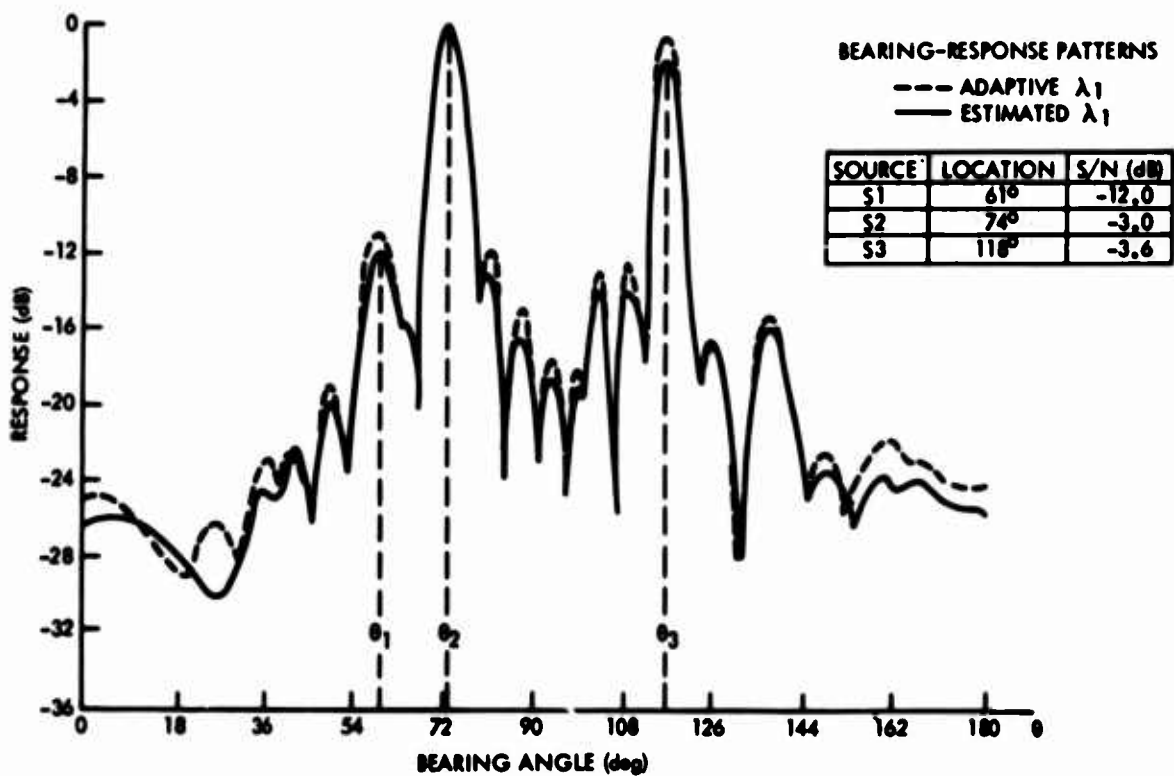


Fig. 7. Adaptive λ_1 Algorithm and Maximum Eigenvalue (Vector) Estimated Bearing-Response Patterns

0.0001 was used. Note that the approximate relative source power levels can be determined by considering the magnitude of the lobe formed in the direction of each source.

Another example of the adaptive source-location processor involves four nonrandom, signal-frequency, moving sources. The same line array as in the examples on page 11 is employed. The total spread in S/N is 20 dB. Identification of the lowest (S/N) source track is still possible, as indicated by Fig. 9. The total processor display time is $T = 60$ seconds for 10,000 iterations. A maximum angular velocity rate for source S1 of 4.5° per second was used. For this example, μ was 0.0005. This value of μ is necessarily larger than in the example given above since the processor had to adapt quite rapidly to follow the moving sources.

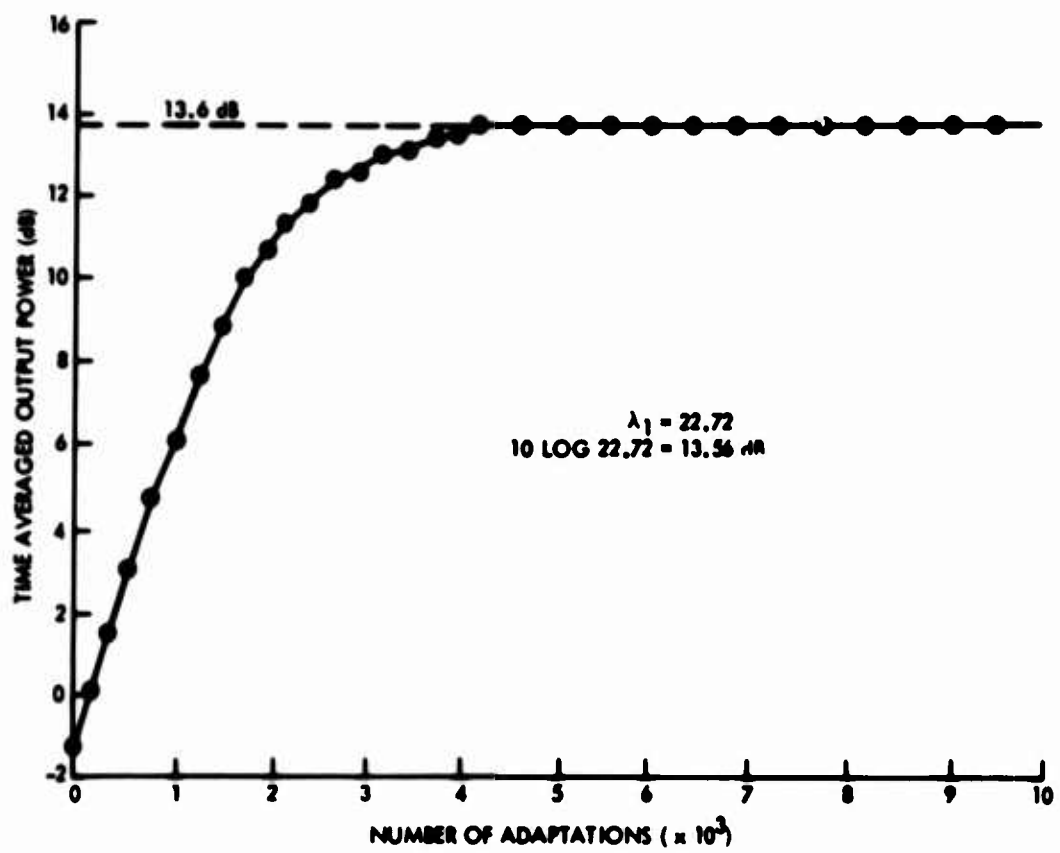
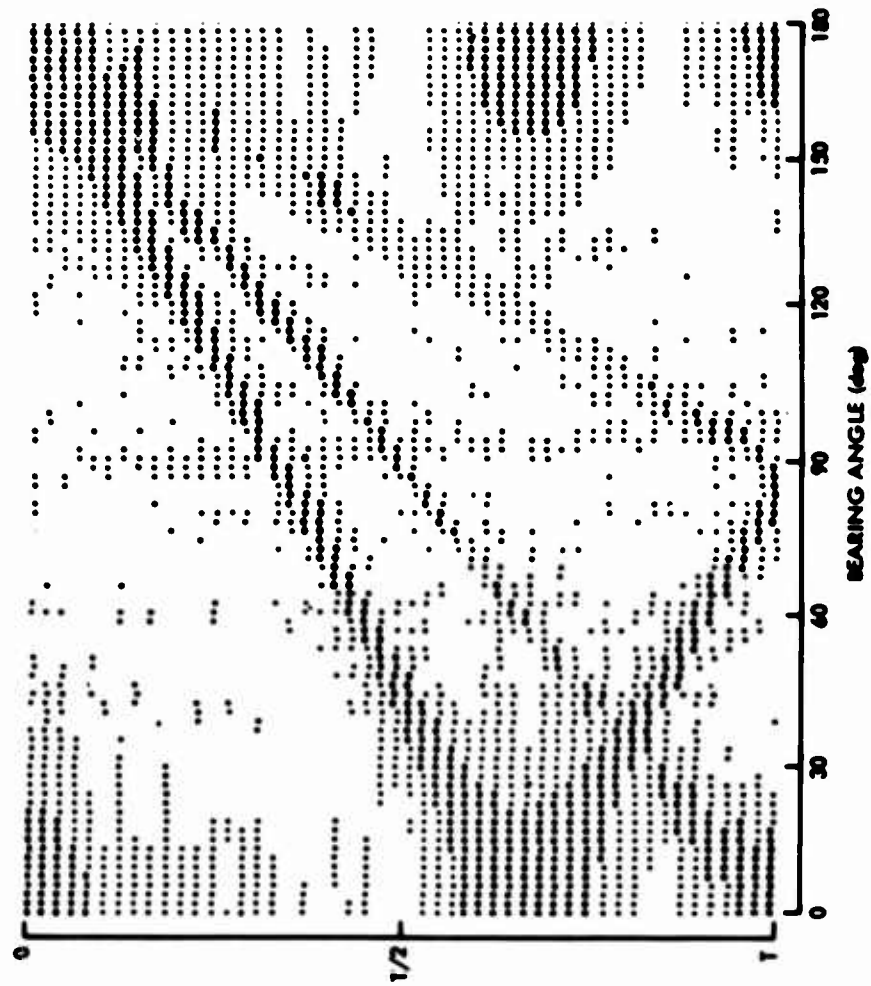


Fig. 8. λ_1 Algorithm Processor Output versus Number of Filter Adaptations



SOURCE	S/N (dB)	TRACK
S1	-3	180° → 0° → 90°
S2	-14	180° → 0°
S3	-17	180° → 90°
S4	-23	90° → 180°

Fig. 9. Bearing Angle Time Display for a Four-Source Signal Field

DISCUSSION

The previous sections have established the background of and proposed an iterative adaptive technique for defining a multiple wavefront spatial field in terms of a series of eigenvector bearing responses. An advantage of this type of spatial array processing over conventional time delay-and-sum beamforming lies in the fact that the eigenvector bearing response to a particular source that is uncorrelated with other sources in the field is invariant with respect to the relative S/N levels for the other sources in the environment. This invariance might also be referred to as a spatial null steering capability for estimating the bearing angle of one source in the presence of interfering sources.²⁸ This null steering effect is observed because of the orthonormality of eigenvectors containing directional information on resolvable wavefronts with statistically uncorrelated complex envelopes.

Another factor in support of this approach is that the number of multichannel filters, such as in Fig. 1, is equal to the number of uncorrelated, resolvable sources that are to be located, rather than the number of directions sampled in the area of the spatial field searched by the array. This would be the case if an optimum, i. e., maximum-likelihood, multisource localization, scheme were implemented. Such a maximum-likelihood processor for estimating the envelope of a wavefront with direction vector \underline{E} would be

$$\underline{H}_{ML} = \Lambda \underline{R}^{-1T} \underline{E}^* \quad (75)$$

where Λ is a scalar given by

$$\Lambda = (\underline{E}^T \underline{R}^{-1T} \underline{E}^*)^{-1} \quad (76)$$

$$= (\underline{E}^{*T} \underline{R}^{-1} \underline{E})^{-1} \quad (77)$$

(See, for example, Reference 28.)

Recalling Eqs. (18) and (23) yields

$$\mathbf{R}^{-1} = \mathbf{M} \boldsymbol{\lambda}^{-1} \mathbf{M}^{\circ T}, \quad (78)$$

$$= \sum_{i=1}^K \frac{1}{\lambda_i} \mathbf{M}_i \mathbf{M}_i^{\circ T}. \quad (79)$$

Substituting Eq. (79) into Eq. (75) gives

$$\mathbf{H}_{ML} = \mathbf{A} \sum_{i=1}^K \frac{a_i}{\lambda_i} \mathbf{M}_i^{\circ}, \quad (80)$$

where

$$a_i = \mathbf{M}_i^T \mathbf{E}^{\circ}. \quad (81)$$

Suppose, that the j th eigenvector (\mathbf{M}_j) has the property

$$a_i = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}, \quad (82)$$

then

$$\mathbf{H}_{ML} = \mathbf{M}_j^{\circ} \quad (83)$$

for uncorrelated wavefront envelopes. Thus, the maximum-likelihood processor is identical to the j th eigenvector processor for processing a wavefront with directional delay vector

$$\mathbf{E} = \mathbf{M}_j. \quad (84)$$

The importance of this result is that if the eigenvector \mathbf{M}_j containing wavefront directional information \mathbf{E} (Eq. (84)) were known, as in the method proposed in this report, then the optimum processor could be specified immediately in the form of Fig. 1, rather than by implementing a matrix filter-beamformer as indicated by Eq. (75).

Implicit in a maximum-likelihood array processor, as conventionally utilized, is an assumption of planar-source generated wavefronts emanating from each spatial search direction. For a physically large, rigid array located in an environment with either curved wavefronts or propagation velocity gradients in the vicinity of the array, the assumption of planar wavefronts could, of course, be troublesome. For nonrigid arrays, relative movement of sensors that can not be as-

certained degrades both conventional and optimal coherent summation for beam formation. No such plane-wave assumption, however, is inherent in the eigenvector decomposition approach to multiple sources location.

A discussion of the numerical accuracy and related probable limitations of the λ_i algorithm (Eq. (66)) is appropriate. Notice that the estimate of the gradient adjustment term (Eq. (67)) for estimating the i th eigenvalue and associated eigenvector depends on the estimates of the $(i - 1)$ larger eigenvalues and associated vectors. Since this gradient estimate does depend on these eigenpair estimates, it is anticipated that the higher the λ_i index the greater the λ_i and M_i estimator variances. This suspected cumulative error effect can be reduced by decreasing the gradient step-size factor μ in Eq. (66), thereby decreasing the M_i estimator variance. However, this approach would have to be balanced with a consideration of the stationarity of the sampled data statistics. This is because μ must also be large enough to allow the algorithms to follow possible time variations in the input statistics. Nonstationarities of this sort, for example, would result from sources with either angular velocity or time-varying spectra.

Finally, all previous considerations have dealt with implementing the λ_i algorithm at a single, discrete frequency. One potential broadband system would consist of eigenvector processors operating in parallel, one for each frequency. The number of narrow-frequency bins of interest would be L . Such a system is illustrated in Fig. 10. The time-sampled sequence $\{X_{in}\}$ from the i th sensor would first be processed by a discrete frequency transform (DFT) or equivalent contiguous narrowband filter bank. The discrete frequency outputs from each sensor DFT are then grouped to form the single frequency (f_q) vector $\underline{X}(n)$, $q = 1, 2, \dots, L$, at the filter update instant t_n . If M resolvable wavefronts (or eigenvectors) are to be detected, then M number of λ_i algorithms are implemented at each frequency. The bearing angle for M wavefronts might then be detected by searching for the peak (or peaks) of the bearing response (Eq. (41)) weighted by the eigenvalue for the i th wavefront at the discrete frequency f_q . This source-bearing selection procedure could be implemented by, first, estimating the broadband bearing angle θ , with the expression

$$B(\theta) = \sum_{q=1}^L \sum_{i=1}^K \lambda_i(f_q) B_i(f_q, \theta), \quad (85)$$

and then choosing the M most prominent bearing angles for $\lambda_i(f_q)$ display.

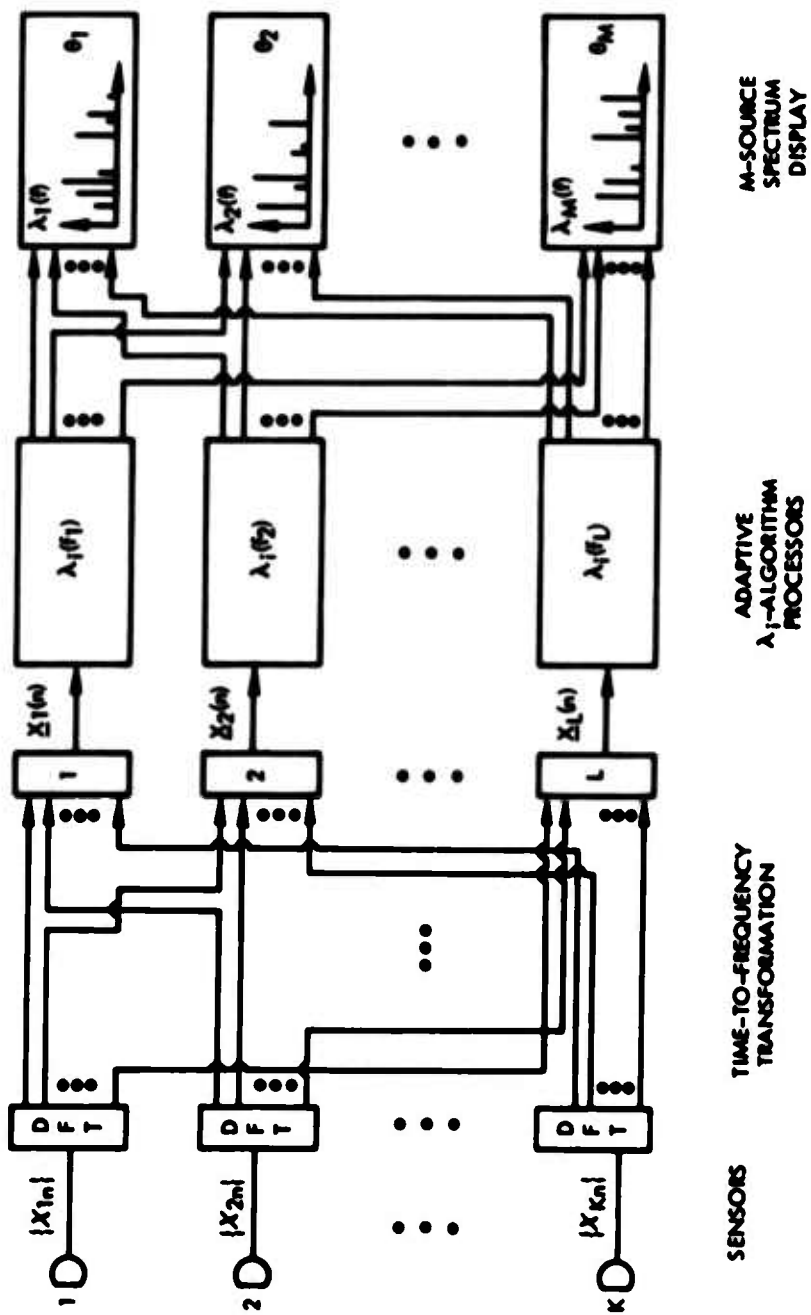


Fig. 10. Broadband λ_i Algorithm Processor for an M-Source Signal Field

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